Adult Basic Education

Mathematics

Mathematics 1104C

Systems of Equations and Inequalities, and Matrices

Curriculum Guide

Prerequisites: Mathematics 1104A, 1104B

Credit Value: 1

Required Mathematics Courses
[Degree and Technical Profile/ Business-Related College Profile]

Mathematics 1104A
Mathematics 1104B
Mathematics 1104C
Mathematics 2104A
Mathematics 2104B
Mathematics 2104C
Mathematics 3104A
Mathematics 3104B
Mathematics 3104C

Table of Contents

To the Instructor

I. Introduction to Mathematics 1104C

The two topics in this course are systems of equations and inequalities, and matrices. Systems of equations in two variables are solved by two methods: graphing and substitution. Although some of the exercises can be completed by using a graphing calculator, it is recommended that students graph several systems by hand to ensure that the process is understood. Students must have the ability to translate word problems into equations. For students who have difficulty in this area, it would be helpful if they could discuss the word problems and compare equations with others. When solving word problems, encourage students to be organized in their approach and to show the steps that they follow.

Students will first graph linear inequalities in two variables and then progress to a system of linear inequalities. Again, students will solve word problems using systems of linear inequalities.

The second topic in this course gives an introduction to matrices. Students will learn that matrices are frequently used to represent real world data. Students will be introduced to the terminology and then matrix calculations: addition, subtraction and multiplication.

II. Prerequisites

To be successful in this course, students should have prerequisite skills to manipulate and graph linear equations. Students must be able to solve equations and translate descriptive statements into equations or inequalities.

III. Textbook

Most of the concepts are introduced, developed and explained in the **Examples**. The instructor must insist that students carefully study and understand each **Example** before moving on to the **Exercises**. In the Study Guide, students are directed to see the instructor if there are any difficulties.

To the Instructor

There are four basic categories included in each section of the textbook which require the student to complete questions:

1. Investigate
2. Discussing the Ideas
3. Exercises
4. Communicating the Ideas

Investigate: This section looks at the thinking behind new concepts. The answers to its questions are found in the back of the text.

Discussing the Ideas: This section requires the student to write a response which clarifies and demonstrates understanding of the concepts introduced. The answers to these questions are not in the student text but are in the *Teacher's Resource Book*. Therefore, in the Study Guide, the student is directed to see the instructor for correction. This will offer the instructor some perspective on the extent of the student's understanding. If necessary, reinforcement or remedial work can be introduced. Students should not be given the answer key for this section as the opportunity to assess the student's understanding is then lost.

Exercises: This section helps the student reinforce understanding of the concepts introduced. There are three levels of **Exercises**:

- **A:** direct application of concepts introduced;
- **B:** multi-step problem solving and some real-life situations;
- **C:** problems of a more challenging nature.

The answers to the **Exercises** questions are found in the back of the text.

Communicating the Ideas: This section helps confirm the student's understanding of a particular lesson by requiring a clearly written explanation. The answers to **Communicating the Ideas** are not in the student text, but are in the *Teacher's Resource Book*. In the Study Guide students are asked to see the instructor for correction.

IV. Technology

It is important that students have a **scientific** calculator and its manual for their individual use. Ensure that the calculator used has the word "scientific" on it as there are calculators designed for calculation in other areas such as business or statistics which would not have the functions needed for study in this area.

To the Instructor

A graphing calculator should be **available** to the students since the text provides many opportunities for its use. The *Teacher's Resource Book* suggests many occasions to utilize a graphing calculator. These suggestions are outlined where there is the heading *Integrating Technology*. In the Study Guide, students are directed to see the instructor when a graphing calculator is required. The *Teacher's Resource Book* contains a module called **Graphing Calculator Handbook** which will help the instructor and student get acquainted with some of the main features of the TI-83 Plus graphing calculator.

Graphing software such as *Graphmatica* or *Winplot* can also be used if the students don't have access to a graphing calculator but do have access to a computer. The textbook doesn't offer the same guidance for graphing with these tools as it does for a graphing calculator but each software program does have a HELP feature to answer questions.

V. Curriculum Guides

Each new ABE Mathematics course has a Curriculum Guide for the instructor and a Study Guide for the student. The Curriculum Guide includes the specific curriculum outcomes for the course. Suggestions for teaching, learning, and assessment are provided to support student achievement of the outcomes. Each course is divided into units. Each unit comprises a **two-page layout of four columns** as illustrated in the figure below. In some cases the four-column spread continues to the next two-page layout.

Curriculum Guide Organization:
The Two-Page, Four-Column Spread

Unit Number - Unit Title	
Outcomes Specific curriculum outcomes for the unit.	**Notes for Teaching and Learning** Suggested activities, elaboration of outcomes, and background information.

Unit Number - Unit Title	
Suggestions for Assessment Suggestions for assessing students' achievement of outcomes.	**Resources** Authorized and recommended resources that address outcomes.

VI. Study Guides

The Study Guide provides the student with the name of the text(s) required for the course and specifies the sections and pages that the student will need to refer to in order to complete the required work for the course. It guides the student through the course by assigning relevant reading and providing questions and/or assigning questions from the text or some other resource. Sometimes it also provides important points for students to note. (See the *To the Student* section of the Study Guide for a more detailed explanation of the use of the Study Guides.) The Study Guides are designed to give students some degree of independence in their work. Instructors should note, however, that there is much material in the Curriculum Guides in the *Notes for Teaching and Learning* and *Suggestions for Assessment* columns that is not included in the Study Guide and instructors will need to review this information and decide how to include it.

VII. Resources

Essential Resources

Addison Wesley Mathematics 11 (Western Canadian edition)
ISBN:0-201-34624-9

Mathematics 11 Teacher's Resource Book (Western Canadian edition) ISBN: 0-201-34626-5

Math 1104C Study Guide

Recommended Resources

*Mathematics 11 Independent Study Guide (*Western Canadian edition) ISBN: 0-201-34625-7

Center for Distance Learning and Innovation: http://www.cdli.ca
Winplot: http://math.exeter.edu/rparris/winplot.html
(Free graphing software)
Graphmatica (Evaluation software available on CD-ROM contained in *Teacher's Resource Book*)
CD Rom accompanying *Teacher's Resource Book*
This CD contains selected solutions from the text and self test solutions from the *Independent Study Guide*.

Other Resources

Math Links: http://mathforum.org

http://www.purplemath.com

http://www.sosmath.com/index.html

http://www.math.com/

http://spot.pcc.edu/~ssimonds/winplot
(Free videos concerning Winplot)

http://www.pearsoned.ca/school/math/math/

VIII. Recommended Evaluation

Written Notes	10%
Assignments	10%
Test(s)	30%
Final Exam *(entire course)*	50%
	100%

The overall pass mark for the course is 50%.

Systems of Equations and Inequalities, and Matrices

Unit 1 - Solving Systems of Equations and Inequalities

Outcomes

1.1 Solve systems of equations graphically.

1.1.1 Define the term *linear system of equations*.

1.1.2 Rearrange equations to express in $y = mx + b$ form.

1.1.3 Given a pair of coordinates, find a system of linear equations for which that point is the solution.

Notes for Teaching and Learning

As students progress through this unit, they will graphically and algebraically solve systems of equations. Students should be encouraged to solve some equations using technology.

The method used often depends on the accuracy required and the equations involved. Although it is recommended that all students graph several systems by hand, the *Teacher's Resource Book* provides many occasions to use the TI-83 calculator. The instructor will find these examples in most sections under the heading *Integrating Technology*.

Students have not been directed in the Study Guide to use a graphing calculator. The instructor, however, should point out to students where it would be a useful tool.

Note: Assign review questions from **Prerequisites** , *Teacher's Resource Book*, Chapter 5, page 4.

The instructor should impress upon the students the importance of using grid paper and drawing neat graphs.

If students have access to graphing calculators, they should be encouraged to use them to complete some exercises in this unit. On pages 308 and 309 in the *Mathematics 11* textbook, there are methods described for using the TI-83 calculator in solving systems of equations.

Suggestions for Assessment

Study Guide questions 1.1 to 1.4 will meet the objectives of Outcome 1.1.

The *Teacher's Resource Book* and the *Independent Study Guide* have extra problems with solutions that could be used for assessment or review.

Resources

Mathematics 11,
Section 5.1,
Solving Systems of Equations by Graphing,
pages 300 - 309

Mathematics 11,
Teacher's Resource Book,
Chapter 5, pages 4 - 8

Mathematics 11,
Independent Study Guide,
page 66

Unit 1 - Solving Systems of Equations and Inequalities

Outcomes

1.2 Solve systems of equations using substitution.

Notes for Teaching and Learning

Note: Assign review questions from **Prerequisites**, *Teacher's Resource Book*, Chapter 5, page 14.

This section deals with solving systems of equations using the method of substitution. The instructor should explain that this method works whether the system is linear or not.

The textbook provides several problems which involve a quadratic equation and a linear equation.

Suggestions for Assessment

Study Guide questions 1.5 and 1.6 will meet the objectives of Outcome 1.2.

Resources

Mathematics 11,
Section 5.4,
Solving Systems by Substitution,
pages 323 - 325

Mathematics 11,
Teacher's Resource Book,
Chapter 5, pages 14 - 16

Mathematics 11,
Independent Study Guide,
page 67

www.cdli.ca Math 3204,
Unit 07, Section 03,
Lesson 03

Unit 1 - Solving Systems of Equations and Inequalities

Outcomes

1.3 Represent and solve problems using linear systems.

Notes for Teaching and Learning

Note: Assign review questions from **Prerequisites**, *Teacher's Resource Book*, Chapter 5, page 19.

Many students have difficulty translating word problems into equations. Since this ability is essential for success in this section, the instructor may need to go to other resources for more practice examples of word problems.

Both **Examples** are solved using addition and subtraction. Students will not learn this method until *Mathematics 2104A*. Therefore, the instructor should guide the students in using substitution to solve these two **Examples**.

Since students will probably find it challenging to write an equation in one variable, the instructor should suggest writing two equations in two variables and then use substitution to find one equation in one variable.

Suggestions for Assessment

Study Guide question 1.7 will meet the objectives of Outcome 1.3.

Resources

Mathematics 11,
Section 5.5,
Problems Involving Linear Systems,
pages 333 - 335

Mathematics 11,
Teacher's Resource Book,
Chapter 5, pages 19 - 21

Mathematics 11,
Independent Study Guide,
pages 67 and 68

Outcomes

1.4 Graph linear inequalities in two variables.

1.4.1 Determine the region represented when given a linear inequality in the form

$y \geq mx + b$ or $y \leq mx + b$.

Notes for Teaching and Learning

Students must be able to graph equations and compare the left and right sides of equations after substituting values for *x* and *y*.

There are exercises in **Prerequisites**, *Teacher's Resource Book*, Chapter 5, page 25 that could be used for review.

This section should give students a solid foundation in linear inequalities before they begin the next section which studies **systems** of linear inequalities.

Students should be able to graph inequalities by hand, but a graphing calculator could be used to check answers at first and later students could use it to graph inequalities.

Masters 5.1 to 5.3 in the *Teacher's Resource Book 11* give instructions for using the TI-83 graphing calculator to graph an inequality.

Students should use the graphing calculator to reproduce the screen shown in **Exercises**, question 3, page 350.

Suggestions for Assessment

Study Guide questions 1.8 and 1.9 will meet the objectives of Outcome 1.4.

Resources

Mathematics 11,
Section 5.7,
Graphing Linear Inequalities in Two Variables,
pages 348 - 351

Mathematics 11,
Teacher's Resource Book,
Chapter 5, pages 25 - 28

Mathematics 11,
Independent Study Guide,
pages 69 and 70

Outcomes

1.5 Solve systems of inequalities graphically.

1.5.1 Determine the system of inequalities that represents a shaded region on a graph.

Notes for Teaching and Learning

If students do not have a solid understanding of graphing inequalities, the instructor should provide review exercises from **Prerequisites**, *Teacher's Resource Book*, Chapter 5, page 29.

In this section, students will solve systems of linear inequalities graphically.

The instructor should guide students when completing questions 1 and 6 in **Exercises** on page 354, because there are no examples of this type included in the student text.

Students should be encouraged to use a TI-83 calculator to reproduce the screens shown in question 2.

Students are <u>not</u> required to determine the area in questions 3 and 4.

Suggestions for Assessment

Study Guide questions 1.10 to 1.12 will meet the objectives of Outcomes 1.5 and 1.6.

The instructor could use *Review*, pages 364 - 365 in the textbook for assessment problems.

Masters 5.6 - 5.10 in the *Teacher's Resource Book* contain a Written Test and a Multiple Choice Test.

The *Teacher's Resource Book* and the *Independent Study Guide* have extra problems with solutions that could be used for assessment or review.

Resources

Mathematics 11,
Section 5.8,
Graphing Systems of Linear Inequalities,
pages 352 - 356, 358 and 359

Mathematics 11,
Teacher's Resource Book,
Chapter 5, pages 29 - 32
Masters 5.6 - 5.10

Mathematics 11,
Independent Study Guide,
pages 70 and 71

Unit 1 - Solving Systems of Equations and Inequalities

Outcomes

1.6 Solve maximum-minimum problems using systems of linear inequalities.

1.6.1 Demonstrate an understanding that the coordinates at the vertex of a particular region on a graph may represent a maximum or minimum value.

Notes for Teaching and Learning

In this section, students will use systems of linear inequalities in applied problems in business when it is necessary to maximize profits or minimize costs.

By this time, students should be aware that a linear inequality is represented by a half-plane on a graph.

The instructor should ensure that students understand that the coordinates at the vertex of a particular region on a graph may represent a maximum or minimum value.

Students have been assigned questions 1 - 3 on page 359. If the instructor sees a need for more work in this area, questions 4 and 5 could also be assigned. If students are to complete question 4 on page 359, they will first have to answer question 10 on page 356.

Suggestions for Assessment

Study Guide questions 1.10 to 1.12 will meet the objectives of Outcomes 1.5 and 1.6.

The instructor could use *Review*, pages 364-365 in the textbook or the *Independent Study Guide* for assessment problems.

Supplementary Examples and Assessing the Outcome in the *Teacher's Resource Book* have extra problems with solutions that could be used for assessment or review.

Masters 5.6 - 5.10 in the *Teacher's Resource Book* contain a Written Test and a Multiple Choice Test.

Resources

Mathematics 11,
Section 5.8,
Graphing Systems of Linear Inequalities,
pages 352 - 356, 358 and 359

Mathematics 11,
Teacher's Resource Book,
Chapter 5, pages 29 - 32
Masters 5.6 - 5.10

Mathematics 11,
Independent Study Guide,
pages 70 and 71

Unit 2 - Matrices

Outcomes

2.1 Demonstrate an understanding of the properties of matrices and apply them.

2.1.1 Develop and apply rules for matrix addition and subtraction.

2.1.2 Develop and apply rules for matrix multiplication.

2.1.3 Develop criteria for determining whether two matrices can be multiplied.

2.1.4 Apply matrix multiplication to a variety of problems.

2.1.5 Use the graphing calculator efficiently for matrix computations.

Notes for Teaching and Learning

This topic, matrices, is **not** covered in the textbook.

The *Appendix* has a workbook which students must use. However, the instructor should find other resources which may be helpful in teaching this topic.

At this level, students will be learning the basic definitions, matrix addition, subtraction and multiplication.

The CDLI site, www.cdli.ca has an interactive Lesson on "using the TI-83 to multiply matrices".

Suggestions for Assessment

Study Guide questions 2.1 to 2.5 will meet the objectives of Outcome 2.1.

Resources

Appendix A
Matrix Workbook

www.sos math.com/matrix

www.cdli.ca , Math 1204, Unit 02,
Section 03, Lesson 02, Lesson 03

www.ed.gov.nl.ca/edu/sp. mathres/mathresources. htm

Appendix A

Matrices

Matrices

A <u>matrix</u> is a rectangular array of numbers within brackets. The array is used to represent real world data and solve real world problems. Simply put, it is a way to arrange data in a table form.

Any table that has rows and columns is a matrix. Databases are examples of matrices used to organize information in matrix form. It is difficult to read through a newspaper and not see examples of matrices. Below is an example.

MJAHL
est. 1967
MARITIME JUNIOR A HOCKEY LEAGUE

MARITIME JUNIOR A HOCKEY LEAGUE
1999-2000
FINAL STANDINGS

MAURICE BENT DIVISION

Team	GP	W	L	T	OL	F	A	P
Halifax	52	38	10	4	2	296	178	82
Antigonish	52	34	13	5	1	273	181	74
Truro	52	32	17	3	0	231	177	67
East Hants	52	8	35	9	1	134	140	26
Amherst	52	8	41	3	1	159	279	19

ROGER MEEK DIVISION

Team	GP	W	L	T	OL	F	A	P
Campbellton	52	37	14	1	1	285	192	76
Summerside	52	24	25	3	1	198	229	52
Charlottetown*	52	19	31	2	0	198	247	40
Moncton	52	18	32	2	1	183	231	40

* - awarded third place on more wins

You should become familiar with the following terms:

- *element* - individual number in a matrix

- *row* - horizontal group of numbers in a matrix

- *column* - vertical group of numbers in a matrix

- *dimension* - number of rows and columns in a matrix. If a matrix has m rows and n columns, it is a ($m \times n$) matrix (pronounce m by n matrix).

- *naming a matrix* - usually given a capital letter, e.g. A, B, X, I

There are different types of matrices.

- Square matrix: has the same number of rows as columns

$$A = \begin{bmatrix} 3 & -1 & 0 \\ 12 & 6 & 4 \\ 5 & 9 & -4 \end{bmatrix}$$ A has dimensions (3 × 3).

- Row matrix: has only one row

$$B = \begin{bmatrix} 5 & 0 & -3 & 8 \end{bmatrix}$$ B is a (1 × 4) matrix. B has 1 row and 4 columns.

- Column matrix: has only one column

$$D = \begin{bmatrix} 2 \\ 6 \\ 0 \\ -4 \end{bmatrix}$$ D is a (4 × 1) matrix. D has 4 rows and 1 column.

- Zero matrix: all elements are zeros

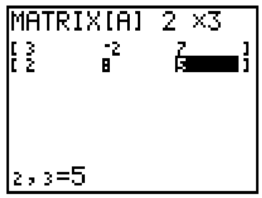
$$L = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$ L is a (2 × 2) matrix.

- Identity matrix: a square matrix with 1's on the main diagonal (top left to bottom right) and all other elements are zeros.

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ I is a (3 × 3) matrix.

You should explore the **matrix** feature on the TI-83 graphing calculator.

Example: If you want to enter the matrix A = $\begin{bmatrix} 3 & -2 & 7 \\ 2 & 8 & 5 \end{bmatrix}$ into the TI-83, use the following steps: **matrix** ▸ **edit** enter the dimensions **2** × **3** then enter the elements in matrix **A.**



1. Given matrix A = $\begin{bmatrix} -1 & 5 & 6 & 8 \\ 3 & 4 & 6 & -2 \\ 12 & 1 & 3 & -1 \end{bmatrix}$

   a) State the dimensions of matrix A.
   b) What is the element in row 2, column 3?
   c) What is the element in row 3, column 4?

2. A store sells two types of sneakers, cross-trainers and court sneakers. In June, the store sold 50 cross-trainers and 30 court sneakers, while in July they sold 80 cross-trainers and 90 court sneakers. Represent this information in a rectangular array (or matrix form). ***Hint***: Let the Rows represent the type of sneaker and let the Columns represent the months (type × month).

3. A music store compared the sales of Rap music CD's to Classical music over 3 months. In November, the store sold 70 Rap CD's and 100 Classical CD's. In December, the sold 120 Rap CD's and 90 Classical CD's. Finally in January there were 80 Rap CD's and 60 Classical CD's sold. Represent this information in matrix form.

4. A store sells three sneaker brands; Nike, Reebok and Adidas. In May, there were 60 Nike, 30 Reebok and 40 Adidas pairs sold. In June, there were 70 Nike, 80 Reebok and 25 Adidas pairs sold. Represent this information in matrix form (brand × month).

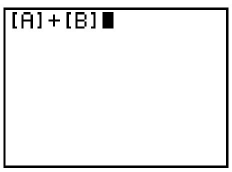
Matrix Addition

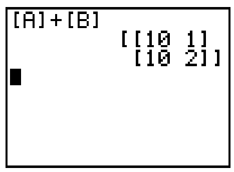
Use a TI-83 to try and discover the rules as to when matrices can be added or subtracted. See your instructor if a graphing calculator is not on hand. Use the problems below to discover the rules.

You should be able to deduce that only elements in matching positions in each matrix can be added. Therefore the matrices <u>*must*</u> *have the same dimensions for them to be added or subtracted.*

Enter the matrices A and B, from question 5a) below, into the TI-83 as shown earlier. Once the matrices have been entered, press **2nd quit** to return to the home screen. To add the matrices:

Press **matrix 1**: **A** press **enter + matrix** ▼ down to **2:B** press **enter**





Note: the same procedure can be followed if matrices are to be multiplied.

5. Use the following problems to complete the table on the next page, if possible. Once the table is completed, look for a pattern and state a rule for matrix addition or subtraction.

a) Find A + B $\quad A = \begin{bmatrix} 6 & -2 \\ 5 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 3 \\ 5 & -2 \end{bmatrix}$

b) Find A + B $\quad A = \begin{bmatrix} 2 & 3 \\ 5 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -6 & 5 \\ -2 & 3 & -4 \end{bmatrix}$

c) Find A − B $\quad A = \begin{bmatrix} 4 & 7 \\ 2 & -1 \\ 0 & 5 \end{bmatrix} \quad B = \begin{bmatrix} -3 & 1 \\ -8 & 4 \\ -5 & 1 \end{bmatrix}$

d) Find A + B $A = \begin{bmatrix} 2 & 0 & -1 \\ 3 & -4 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 \\ 5 & 2 \end{bmatrix}$

e) Find A + B $A = \begin{bmatrix} 2 & 1 \\ 5 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 4 \end{bmatrix}$

f) Find A − B $A = \begin{bmatrix} 4 & 6 & -8 \\ -2 & 5 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 18 & 3 & 12 \\ 1 & 0 & 5 \end{bmatrix}$

g) Complete the table:

Dimensions of matrix A	Dimensions of matrix B	Dimensions of answer
a)		
b)		
c)		
d)		
e)		
f)		

6. Simplify: (Use paper and pencil and the rule you developed in Question 5. DO NOT use a graphing calculator.)

a) $\begin{bmatrix} 4 & 1 \\ -2 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 3 \\ 2 & 3 \end{bmatrix}$

b) $\begin{bmatrix} -2 & -3 \\ -2 & 10 \end{bmatrix} + \begin{bmatrix} -2 & 3 \\ 2 & -8 \end{bmatrix}$

c) $\begin{bmatrix} 3 & 61 \\ 23 & 14 \end{bmatrix} + \begin{bmatrix} 0 & -2 \\ 6 & 4 \end{bmatrix}$

d) $\begin{bmatrix} 12 & 18 \\ 21 & -14 \end{bmatrix} + \begin{bmatrix} 4 & 6 \\ -3 & -6 \end{bmatrix}$

e) $\begin{bmatrix} 5 & 1 \\ -4 & 7 \end{bmatrix} + \begin{bmatrix} 8 & 2 \\ 4 & -5 \end{bmatrix}$

f) $\begin{bmatrix} 11 & 1 \\ 16 & -4 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ 4 & -6 \end{bmatrix}$

g) $\begin{bmatrix} 1 & 2 \\ -3 & 5 \\ 4 & 6 \end{bmatrix} + \begin{bmatrix} 0 & 5 \\ 2 & 4 \\ 3 & -2 \end{bmatrix}$

h) $\begin{bmatrix} 0 & -2 & 4 \\ 3 & -2 & -6 \end{bmatrix} + \begin{bmatrix} 1 & -5 & 0 \\ 3 & -6 & -2 \end{bmatrix}$

i) $\begin{bmatrix} 1 & 5 & 3 \\ 2 & 7 & -1 \end{bmatrix} + \begin{bmatrix} 5 & 7 & 1 \\ 8 & -6 & 11 \end{bmatrix}$

j) $\begin{bmatrix} 7 & 5 & 2 \\ 9 & 1 & 0 \\ 3 & 6 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 2 & 8 \\ 1 & 0 & 6 \\ 9 & 11 & -2 \end{bmatrix}$

Multiplication

Scalar Multiplication

7. If A = $\begin{bmatrix} 4 & 3 \\ -1 & 0 \end{bmatrix}$, find A + A.

$$A + A = \begin{bmatrix} 4 & 3 \\ -1 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ -1 & 0 \end{bmatrix} =$$

Now, find 2A.

$$2A = 2\begin{bmatrix} 4 & 3 \\ -1 & 0 \end{bmatrix} =$$

What do you notice?

8. With the matrix below, evaluate 3A.

$$A = \begin{bmatrix} 3 & -2 \\ 5 & 1 \end{bmatrix}$$

9. A store sells two types of sneakers, cross-trainers and court sneakers. In June, the store sold 50 cross-trainers and 30 court sneakers, while in July they sold 80 cross-trainers and 90 court sneakers. This information is represented in matrix form below. If sales were twice the original projections, represent this solution in matrix form.

$$A = \begin{array}{c} \\ \text{Trainers} \\ \text{Court} \end{array} \begin{array}{c} \begin{array}{cc} \text{June} & \text{July} \end{array} \\ \begin{bmatrix} 50 & 80 \\ 30 & 90 \end{bmatrix} \end{array}$$

10. Does scalar multiplication change the dimensions of a matrix? Give examples in your answer.

11. If $J = \begin{bmatrix} 5 & 3 & 1 \\ 1 & 2 & 0 \end{bmatrix}$, $F = \begin{bmatrix} 1 & -4 & 3 \\ 6 & 1 & 5 \end{bmatrix}$, and $M = \begin{bmatrix} 4 & 5 & -7 \\ 2 & 3 & 8 \end{bmatrix}$,

evaluate the following:

a) J − M b) 2J + F c) 3F + 2M d) J − F + 2M

Zero Matrix

For the addition of matrices, one special matrix, the zero matrix, plays a role similar to the number zero.

For example, $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$

and for scalar multiplication, $x\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

12. If $K = \begin{bmatrix} 3 & 4 \\ 2 & 1 \\ 0 & 9 \end{bmatrix}$ and $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$, evaluate the following:

a) K + A b) 3A

Matrix Multiplication

Example 1: Read and complete the accompanying exercises. Formulate a rule for matrix multiplication.

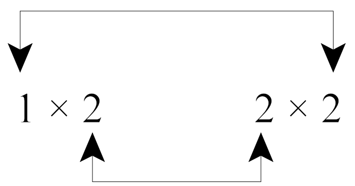
Multiply A × B.

$A = \begin{bmatrix} 1 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} -4 & 2 \\ 5 & 1 \end{bmatrix}$

First, we must see if these matrices can be multiplied. Write the dimensions of each.

<u>Matrix A</u> — row × column

<u>Matrix B</u> — row × column



Look at the given dimensions (1 × 2) and (2 × 2).
If the two <u>inside</u> numbers are the <u>same</u> then the matrices <u>can be multiplied</u>. The *result* will be a matrix with dimensions determined by the *outer* numbers. For the above example, the inner numbers are both 2 and thus multiplication can be done. The outer numbers are 1 and 2 and thus dimensions of the solution matrix is (1 × 2) (1 row and 2 columns).

$\begin{bmatrix} \_ & \_ \end{bmatrix}$

To fill in these blanks, name their positions. The first blank is in the **1st row**, **1st column** position. To get the element that goes in this blank multiply the elements in the **1st row** of matrix A by the elements in the **1st column** of matrix B.
ie: **(1 × −4) + (5 × 5) = −4 + 25 = 21**. The first blank is the element **21.**

The second blank has the position, **1st row 2nd column**. The element that goes here comes from multiplying the elements in the **1st row** of matrix A and the **2nd column** of matrix B.
ie**:** **(1 × 2) + (5 × 1) = 7**. The second blank is the element **7**.

$A \times B = \begin{bmatrix} 21 & 7 \end{bmatrix}$

Example 2:

If P = $\begin{bmatrix} 4 & 1 \\ 0 & -2 \end{bmatrix}$ and Q = $\begin{bmatrix} 0 & -4 & 3 \\ 1 & 5 & 2 \end{bmatrix}$ find P × Q.

First, check the dimensions of P and Q and decide whether it is possible to multiply them, and if so, what the dimensions of the product matrix will be.

Matrix P	Matrix Q
row × column	row × column
2 × 2	2 × 3

The two *inside* numbers are the same, therefore the matrices can be multiplied. The two *outside* numbers are 2 and 3, therefore the dimensions of the product matrix is (2 × 3).

P × Q = $\begin{bmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \end{bmatrix}$

The first blank is in the **1st row**, **1st column** position. Therefore, you must multiply the elements in the **1st row** of matrix P by the elements in the **1st column** of matrix Q.

ie: (4 × 0) + (1 × 1) = 1. The first blank is the element 1.

P × Q = $\begin{bmatrix} \underline{1} & \_ & \_ \\ \_ & \_ & \_ \end{bmatrix}$

The element which goes in the **1st row** and the **2nd column** position is found by multiplying the elements in the **1st row** of matrix P and the **2nd column** of matrix Q.

ie: (4 × −4) × (1 × 5) = −16 + 5 = −11

P × Q = $\begin{bmatrix} 1 & -11 & \_ \\ \_ & \_ & \_ \end{bmatrix}$

Similarly: the element in the **1st row**, **3rd column** is found by multiplying the elements in **row 1** of matrix P by **column 3** of matrix Q.
ie: (4 × 3) + (1 × 2) = 12 + 2 = 14

$$P \times Q = \begin{bmatrix} 1 & -11 & 14 \\ \_ & \_ & \_ \end{bmatrix}$$

The element in the **2nd row**, **1st column** is found by multiplying the elements in the **2nd row** of matrix P, and the **1st** column of matrix Q.

ie: (0 × 0) + (−2 × 1) = −2

$$P \times Q = \begin{bmatrix} 1 & -11 & 14 \\ -2 & \_ & \_ \end{bmatrix}$$

Using the pattern established above, complete the other two blanks in the table.

Example 3: If $L = \begin{bmatrix} 1 & 4 & 0 \\ 9 & -3 & -1 \end{bmatrix}$ and $M = \begin{bmatrix} 4 & 1 & 9 \\ 0 & 9 & 4 \end{bmatrix}$

Find L × M.

First, check dimensions of L and M and decide whether it is possible to multiply them.

Matrix L	Matrix M
row × column	row × column
2 × 3	2 × 3

The two *inside* numbers are *different*: 2 and 3, therefore the matrices cannot be multiplied.
End of problem!

13. Use the following problems to complete the accompanying table. When possible, find the solution for A × B. Use paper and pencil. You may use your graphing calculator to check your answers.

a) $A = \begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix}$ $B = \begin{bmatrix} -2 & 0 \\ 7 & -3 \end{bmatrix}$ b) $A = \begin{bmatrix} 1 & 5 \end{bmatrix}$ $B = \begin{bmatrix} -4 & 2 \\ 5 & 1 \end{bmatrix}$

c) $A = \begin{bmatrix} 4 & -6 \\ 3 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 3 & -4 \end{bmatrix}$ d) $A = \begin{bmatrix} 3 & 1 \\ 4 & 7 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 9 \\ 6 & -1 \\ 0 & 0 \end{bmatrix}$

e) $A = \begin{bmatrix} 0 & 3 \\ 5 & -1 \end{bmatrix}$ $B = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ f) $A = \begin{bmatrix} 8 \\ -4 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 4 \\ 3 & -7 \end{bmatrix}$

g) $A = \begin{bmatrix} 6 & 0 & -2 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 1 & -2 \\ -3 & 6 & -1 \\ 3 & 2 & 7 \end{bmatrix}$

Dimensions of A	Dimensions of B	Dimensions of A × B
a)		
b)		
c)		
d)		
e)		
f)		
g)		

14. In the above series of problems, find the solution matrix for **B × A**. Are the answers the same as those for **A × B**? What does this tell you about matrix multiplication?

Example

A store sells three sneaker brands: Nike, Reebok and Adidas. In May there were 60 Nike, 30 Reebok and 40 Adidas pairs sold. In June there were 70 Nike, 80 Reebok and 25 Adidas pairs sold. Represent this information in matrix form (brand × month). The price of the sneakers is Nike $90, Reebok $70, and Adidas $85. Write this in matrix form (price × brand). Finally multiply these matrices to determine the revenue generated each month.

<u>Solution</u>:

$$\overset{\quad N \quad\; R \quad\; A}{\begin{bmatrix} 90 & 70 & 85 \end{bmatrix}} \overset{\text{May} \quad \text{June}}{\begin{bmatrix} 60 & 70 \\ 30 & 80 \\ 40 & 25 \end{bmatrix}}$$

Price × Brand **Brand × Month**

$$= \begin{bmatrix} 10{,}900 & 14{,}025 \end{bmatrix}$$

This matrix tells us that $10,900 was generated by all brands in May and $14,025 in June.

15. Two outlets of an electronics store sell 3 comparable items. Use matrix multiplication to show the total revenue that these items could generate in each store when they are sold at the regular price and at the sale price.

Number of items in each store

	TV's	Stereos	Cameras
Carbonear	85	100	60
Pasadena	70	120	90

Prices of items

	Regular	Sale Price
TV's	$450	$300
Stereos	$320	$250
Cameras	$280	$170

16. a) Write the following information from the CFL in matrix form and label it matrix A.

	W	L	T
Toronto	6	1	1
Montreal	5	2	1
B.C.	3	3	2
Calgary	2	6	0

Matrix B represents the points awarded for a win, a loss and a tie.

$$\textbf{B} = \begin{matrix} \textbf{W} \\ \textbf{L} \\ \textbf{T} \end{matrix} \overset{\textbf{Points}}{\begin{bmatrix} \textbf{2} \\ \textbf{0} \\ \textbf{1} \end{bmatrix}}$$

b) What are the dimensions of matrix A and matrix B?

c) Calculate **A × B**.

d) What do the elements in the product matrix represent?

17. Your teacher keeps a record of your marks in matrix form with rows representing students and columns representing the test results in %. Class tests/assignments are worth 60% of the term mark while the final exam is worth the remaining 40%. There are five class tests worth 60% ÷ 5 = 12% (.12) each. The final exam is worth 40% (.40).

a) Enter the information below into matrix **A** in the TI-83.

b) Create matrix **B** (6 × 1) representing the values of the tests.
Enter this into matrix **B** in the TI-83.

c) Calculate **A** × **B**.

d) What do the elements in the product matrix represent?

	#1	#2	#3	#4	#5	Final
Anderson, N.	75	59	88	79	91	85
Balcom, P.	53	49	62	59	70	60
Davis, T.	63	82	84	76	89	92
Hunt, S.	92	94	90	89	95	96
Noonan, L.	83	76	87	83	55	62

18. Multiply. Use paper and pencil first. Check your answers on your TI-83.

a) $\begin{bmatrix} 3 & 4 \\ -1 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 3 \\ -1 & 5 \end{bmatrix} \times \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$

c) $\begin{bmatrix} 2 & -4 \\ 1 & 9 \end{bmatrix} \times \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}$

d) $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 4 & 6 \\ -3 & -6 \end{bmatrix}$

e) $\begin{bmatrix} 3 & -2 \\ 5 & -4 \end{bmatrix} \times \begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix}$

f) $\begin{bmatrix} 1 & 2 \\ -3 & 5 \\ 4 & 6 \end{bmatrix} \times \begin{bmatrix} 0 & 5 \\ 2 & 4 \\ 3 & -2 \end{bmatrix}$

g) $\begin{bmatrix} 0 & -2 & 4 \\ 3 & -2 & -6 \end{bmatrix} \times \begin{bmatrix} 1 & -5 & 0 \\ 3 & -6 & -2 \end{bmatrix}$

h) $\begin{bmatrix} 1 & 2 \\ -3 & 5 \end{bmatrix} \times \begin{bmatrix} 2 & 3 & -4 \\ 1 & 2 & 1 \end{bmatrix}$

i) $\begin{bmatrix} 2 & 5 \end{bmatrix} \times \begin{bmatrix} 1 & -3 \\ 4 & 2 \end{bmatrix}$

j) $\begin{bmatrix} 7 & 5 & 2 \\ 9 & 1 & 0 \\ 3 & 6 & 4 \end{bmatrix} \times \begin{bmatrix} 5 & 2 & 8 \\ 1 & 0 & 6 \\ 9 & 11 & -2 \end{bmatrix}$

Answers for Selected Problems

1. a) 3×4
   b) 6
   c) -1

2.

$$A = \begin{array}{c} \\ \text{Trainers} \\ \text{Court} \end{array} \begin{array}{c} \begin{array}{cc} \text{June} & \text{July} \end{array} \\ \begin{bmatrix} 50 & 80 \\ 30 & 90 \end{bmatrix} \end{array}$$

3

$$B = \begin{array}{c} \\ \text{Rap} \\ \text{Classical} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Nov} & \text{Dec} & \text{Jan} \end{array} \\ \begin{bmatrix} 70 & 120 & 80 \\ 100 & 90 & 60 \end{bmatrix} \end{array}$$

4.

$$A = \begin{array}{c} \\ \text{Nike} \\ \text{Reebok} \\ \text{Adidas} \end{array} \begin{array}{c} \begin{array}{cc} \text{May} & \text{June} \end{array} \\ \begin{bmatrix} 60 & 70 \\ 30 & 80 \\ 40 & 25 \end{bmatrix} \end{array}$$

5. a) $\begin{bmatrix} 10 & 1 \\ 10 & 2 \end{bmatrix}$

   b) Cannot be added because dimensions are different.

c) $\begin{bmatrix} 7 & 6 \\ 10 & -5 \\ 5 & 4 \end{bmatrix}$

d) Cannot be added.

e) Cannot be added.

f) $\begin{bmatrix} -14 & 3 & -20 \\ -3 & 5 & -5 \end{bmatrix}$

6.

a) $\begin{bmatrix} 4 & 4 \\ 0 & 3 \end{bmatrix}$

b) $\begin{bmatrix} -4 & 0 \\ 0 & 2 \end{bmatrix}$

c) $\begin{bmatrix} 3 & 59 \\ 29 & 18 \end{bmatrix}$

d) $\begin{bmatrix} 16 & 24 \\ 18 & -20 \end{bmatrix}$

e) $\begin{bmatrix} 13 & 3 \\ 0 & 2 \end{bmatrix}$

f) $\begin{bmatrix} 14 & 0 \\ 20 & -10 \end{bmatrix}$

g) $\begin{bmatrix} 1 & 7 \\ -1 & 9 \\ 7 & 4 \end{bmatrix}$

h) $\begin{bmatrix} 1 & -7 & 4 \\ 6 & -8 & -8 \end{bmatrix}$

i) $\begin{bmatrix} 6 & 12 & 4 \\ 10 & 1 & 10 \end{bmatrix}$

j) $\begin{bmatrix} 12 & 7 & 10 \\ 10 & 1 & 6 \\ 12 & 17 & 2 \end{bmatrix}$

7. $A + A = \begin{bmatrix} 8 & 6 \\ -2 & 0 \end{bmatrix}$ $2A = \begin{bmatrix} 8 & 6 \\ -2 & 0 \end{bmatrix}$

8. $3A = \begin{bmatrix} 9 & -6 \\ 15 & 3 \end{bmatrix}$

9.

$$2A = 2 \times \begin{array}{c} \\ \text{Trainers} \\ \text{Court} \end{array} \begin{array}{c} \begin{array}{cc} \text{June} & \text{July} \end{array} \\ \begin{bmatrix} 50 & 80 \\ 30 & 90 \end{bmatrix} \end{array} = \begin{array}{c} \begin{array}{cc} \text{June} & \text{July} \end{array} \\ \begin{bmatrix} 100 & 160 \\ 60 & 180 \end{bmatrix} \end{array}$$

11. a) $\begin{bmatrix} 1 & -2 & 8 \\ -1 & -1 & -8 \end{bmatrix}$ b) $\begin{bmatrix} 11 & 2 & 5 \\ 8 & 5 & 5 \end{bmatrix}$

c) $\begin{bmatrix} 11 & -2 & -5 \\ 22 & 9 & 31 \end{bmatrix}$ d) $\begin{bmatrix} 12 & 17 & -16 \\ -1 & 7 & 11 \end{bmatrix}$

12. a) $\begin{bmatrix} 3 & 4 \\ 2 & 1 \\ 0 & 9 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

13. a) $\begin{bmatrix} -11 & 3 \\ 29 & -9 \end{bmatrix}$ b) $\begin{bmatrix} 21 & 7 \end{bmatrix}$ c) Cannot be multiplied d) Cannot be multiplied

e) $\begin{bmatrix} 9 \\ -8 \end{bmatrix}$ f) Cannot be multiplied g) $\begin{bmatrix} 24 & 2 & -26 \end{bmatrix}$

15.

$$N = \begin{array}{l} \text{Carbonear} \\ \text{Pasadena} \end{array} \overset{\begin{array}{ccc} \text{TV's} & \text{Stereos} & \text{Cameras} \end{array}}{\begin{bmatrix} 85 & 100 & 60 \\ 70 & 120 & 90 \end{bmatrix}}$$

$$P = \begin{array}{r} \text{TV's} \\ \text{Stereos} \\ \text{Cameras} \end{array} \overset{\begin{array}{cc} \text{Regular} & \text{Sales} \end{array}}{\begin{bmatrix} \$450 & \$300 \\ \$320 & \$250 \\ \$280 & \$170 \end{bmatrix}}$$

$$N \times P = \begin{bmatrix} \$87{,}050 & \$60{,}700 \\ \$95{,}100 & \$66{,}300 \end{bmatrix}$$

16.

c)

$$A = \begin{array}{r} \textbf{Toronto} \\ \textbf{Montreal} \\ \textbf{B. C.} \\ \textbf{Calgary} \end{array} \overset{\begin{array}{ccc} \textbf{W} & \textbf{L} & \textbf{T} \end{array}}{\begin{bmatrix} 6 & 1 & 1 \\ 5 & 2 & 1 \\ 3 & 3 & 2 \\ 2 & 6 & 0 \end{bmatrix}}$$

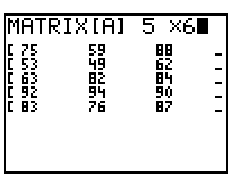
Points

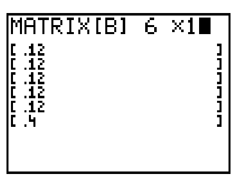
$$\mathbf{B} = \begin{array}{c} \textbf{W} \\ \textbf{L} \\ \textbf{T} \end{array} \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

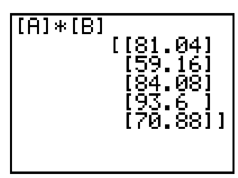
$$A \times B = \begin{bmatrix} 13 \\ 11 \\ 8 \\ 4 \end{bmatrix}$$

d) Total points for each team.

17.c)







d) Mark for each student.

18. a) $\begin{bmatrix} 5 & 22 \\ -4 & 2 \end{bmatrix}$ b) $\begin{bmatrix} 9 & 11 \\ 7 & 21 \end{bmatrix}$ c) $\begin{bmatrix} 10 & -20 \\ -6 & 34 \end{bmatrix}$ d) $\begin{bmatrix} 5 & 6 \\ 0 & -6 \end{bmatrix}$

e) $\begin{bmatrix} -1 & -17 \\ -3 & -31 \end{bmatrix}$ f) no solution g) no solution h) $\begin{bmatrix} 4 & 7 & -2 \\ -1 & 1 & 17 \end{bmatrix}$

i) $\begin{bmatrix} 22 & 4 \end{bmatrix}$ j) $\begin{bmatrix} 58 & 36 & 82 \\ 46 & 18 & 78 \\ 57 & 50 & 52 \end{bmatrix}$