

Adult Basic Education (ABE)

Level III Mathematics

Mathematics 3101C

Logarithmic Functions/Sinusoidal Functions/Borrowing Money Curriculum Guide

Resource: *Principles of Mathematics 12. Nelson. 2012. ISBN-13: 978-0-17-654038-8*

Level III Degree and Technical/Business-Related College Profiles Mathematics Courses (Academic)

Mathematics 1101A: Measurement/Trigonometry/Factors and Products

Mathematics 1101B: Roots and Powers/Relations and Functions

Mathematics 1101C: Linear Functions/Systems of Linear Equations

Mathematics 2101A: Reasoning/Angles and Triangles/Trigonometry

Mathematics 2101B: Radicals/Statistics/Quadratic Functions

Mathematics 2101C: Quadratic Equations/Proportional Reasoning

Mathematics 3101A: Set Theory/Counting Methods/Probability

Mathematics 3101B: Rational Expressions and Equations/Polynomial Functions/Exponential Functions

Mathematics 3101C: Logarithmic Functions/Sinusoidal Functions/Borrowing Money



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General Information

Introduction

Mathematics 3101B when completed with **Mathematics 3101A** and **B** is equivalent to the Newfoundland and Labrador senior high school **Mathematics 3201 (Academic)** course.

Pre-requisite

Students must have passed **Mathematics 3101B**.

Resources

The student resource for this course is:

- *Principles of Mathematics 12. Nelson. 2012. ISBN-13: 978-0-17-654038-8.*

The instructor resources for this course are:

- *Principles of Mathematics 12 Teacher's Resource. Nelson. 2012. ISBN-13: 978-0-17-654044-9.*
- *Principles of Mathematics 12 Teacher's Resource CD-ROM.*

Instructors may also supplement with other resources at their discretion.

Study Guide

The Study Guide provides the student with Required Work for the course. It guides the student through the course by assigning relevant reading and exercises from the student resource. Sometimes the Study Guide provides important points for students to think about, to remember or to note. The Study Guide is designed to give students some degree of independence in their work. There is information in the Curriculum Guide applicable to teaching, learning and assessment that is not included in the Study Guide. Instructors should review this information and decide how to use it when teaching students.

Instructors can also exercise professional judgment and make minor alterations to the Required Work in the Study Guide. For example, an instructor may decide that it is unnecessary to assign students all the exercises to complete within each lesson.

Curriculum Guide

The Curriculum Guide includes the specific curriculum outcomes and achievement indicators for the course. The specific curriculum outcomes are listed numerically, and the achievement indicators are listed alphabetically. Suggestions for teaching, learning and assessment are also provided to support student achievement of the outcomes. Some of these suggestions will also be repeated in the curriculum guides for other mathematics courses as appropriate. The curriculum guide also states the pre-requisite for each Level III mathematics course.

Mathematics 3101C Outcomes/Achievement Indicators

Unit 1: Logarithmic Functions

1. Represent data, using exponential and logarithmic functions, to solve problems.
 - a) Describe, orally and in written form, the characteristics of a logarithmic function by analyzing its graph.
 - b) Describe, orally and in written form, characteristics of a logarithmic function by analyzing its equation.
 - c) Match equations in a given set to their corresponding graphs.
2. Demonstrate an understanding of logarithms and the law of logarithms.
 - a) Express a logarithmic equation as an exponential equation and vice versa.
 - b) Determine the value of a logarithmic expression, such as $\log_2 8$, without technology.
 - c) Develop the laws of logarithms, using numerical examples and exponent laws.
 - d) Determine an equivalent expression for a logarithmic expression by applying the laws of logarithms.
3. Solve problems that involve exponential equations.
 - a) Solve problems that involve logarithmic scales, such as Richter scale and the pH scale.
 - b) Determine the solution of an exponential equation in which bases are not powers of one another; e.g., $2^{x-1} = 3^{x+1}$.
 - c) Determine the approximate value of a logarithmic expression, such as $\log_2 9$, with technology.
 - d) Solve problems that involve the application of exponential equations.
 - e) Graph data, and determine the logarithmic function that best approximates the data.
 - f) Interpret the graph of a logarithmic function that models a situation, and explain the reasoning.

g) Solve, using technology, a contextual problem that involves data that is best represented by graphs of logarithmic functions and explain the reasoning.

Unit 2: Sinusoidal Functions

1. Represent data using sinusoidal functions to solve problems.
 - a) Demonstrate an understanding of angles expressed in degrees and radians.
 - b) Describe orally and in written form, the characteristics of a sinusoidal function by analyzing its graph.
 - c) Interpret the graph of a sinusoidal function that models a situation, and explain the reasoning.
 - d) Describe, orally and in written form, the characteristics of a sinusoidal function by analyzing its equation.
 - e) Match equations in a given set to their corresponding graphs.
 - f) Graph data, and determine the sinusoidal function that best approximates the data.
 - g) Solve, using technology, a contextual problem that involves data that is best represented by graphs of sinusoidal functions, and explain the reasoning.

Unit 3: Financial Mathematics: Borrowing Money

1. Solve problems that involve compound interest in financial decision making.
 - a) Explain the advantages and disadvantages of compound interest and simple interest.
 - b) Identify situations that involve compound interest.
 - c) Solve a contextual problem that involves compound interest.
 - d) Compare, in a given situation, the total interest paid or earned for different compounding periods.
 - e) Determine the total interest of a loan given the principal, interest rate and a number of compounding periods.
 - f) Determine, using technology, the total cost of a loan under a variety of conditions; e.g., different amortization periods, interest rates, compounding periods and terms.
 - g) Determine using technology, the unknown variable in compound interest loan situations.
 - h) Compare and explain, using technology, different credit options that involve compound interest, including bank and store credit cards and special promotions.
2. Analyze costs and benefits of renting, leasing and buying.
 - a) Identify and describe examples of assets that appreciate or depreciate.
 - b) Compare, using examples, renting, leasing and buying.
 - c) Justify, for a specific set of circumstances, if renting, buying or leasing would be advantageous.
 - d) Solve, using technology, a contextual problem that involves cost-and-benefit analysis.

Recommended Evaluation

Written Notes (Including all the Required Work)	10%
Assignments	20%
Tests	20%
Final Exam (entire course)	50%
Total	100%

Instructors have the discretion to make minor changes to this evaluation scheme.

Unit 1: Logarithmic Functions—Suggestions for Teaching and Learning

- In this unit, students will be introduced to logarithms, how to represent them, and how to use them to model and solve problems. Students will also use the laws of logarithms to simplify and evaluate expressions.
- Students will also use graphing technology to enter data and create a model using a logarithmic regression.
- Discuss a comparison of the graphs of exponential and logarithmic functions, and identify characteristics with emphasis on domain and range, intercepts and end behavior.
- Discuss a comparison of the graphs of exponential and logarithmic functions as reflections about the line $y=x$.
- It may be beneficial for students to first investigate the characteristics of logarithmic functions with base 10. Using a graphing calculator, ask students to graph $y=10^x$ and the line $y=x$.
- Ensure students understand how to graph logarithmic functions with base 10 and base e .
- Discuss with students that the graph of the logarithmic curve ($y=\log_{10}x$) is a reflection of the exponential curve ($y=10^x$).
- Discuss with students the effect of changing the value of “ a ” by comparing the logarithm functions $y=\log_{10}x$ and $y=a\log_{10}x$, where $a\neq 0$.
- Students should understand that when $a>0$, the y -values increase as the x -values increase. This is an increasing function from quadrant IV to quadrant I. If $a<0$, y -values decrease as the x -values increase. This is a decreasing function from quadrant I to quadrant IV.
- Discuss with students the natural logarithmic function $y=\log_e x$ ($y=\ln x$) and the exponential function $y=e^x$. Note that the value of e is an irrational number.
- Ensure that students understand how to match equations of logarithmic and exponential functions in a given set to their corresponding graphs.
- Ensure students are able to solve problems involving logarithmic scales such as the Richter scale, pH scale and the decibel scale.
- Ensure that students can determine the exact or approximate solution regardless of the base of the logarithmic expression.

Unit 1: Logarithmic Functions—Suggestions for Assessment

- Instructors can use the BLM's on the CD-ROM to further reinforce the unit concepts.
- The BLM's on the CD-ROM can be useful for developing unit tests and the final exam.
- Instructors have discretion to combine the last unit test with the final exam if beneficial to the student.
- Students must pass the final exam with a minimum grade of 50% to receive credit for this course.
- Instructors should encourage students to reflect on the math concepts in this unit to relate to everyday life.
- Instructors should engage students in discussions to verbalize student thinking on the math concepts.
- Instructors should require students to always show complete calculations with correct units when relevant.
- Instructors can use their own professional judgment to design assessment tools (additional exercises, word problems, assignments, reflections, math journals, etc.) to meet individual student needs.

Unit 2: Sinusoidal Functions—Suggestions for Teaching and Learning

- In this unit, students investigate sinusoidal functions. They will study $y=\sin(x)$ and $y=\cos(x)$, and the base graph will be developed in degrees and then radians. Students will describe features such as amplitude, period, midline, domain, range, vertical and horizontal shifts. Using technology, students will create scatter plots for data and use regression to generate modelling functions.
- Students will examine characteristics of sinusoidal functions by analyzing the graph and its corresponding equation: $y=a\sin b(x-c)+d$ and $y=a\cos b(x-c)+d$.
- Introduce students to radian measure as alternative way to express the size of an angle.
- Ensure that students are able to change a degree measure to radian measure and each radian measure to degree measure.
- Ensure that students understand that 2π radians = 360° , π radians = 180° , $\pi/2$ radians = 90° , etc.
- Discuss with students the main advantages of using radian measure with sinusoidal functions. The advantage is that radians are directly related to the radius of a circle.
- Discuss with students patterns in the sine and cosine ratios. Use graphing technology and a table to record the points. Choose an approximate increment for Θ when graphing $y=\sin(\Theta)$ and $y=\cos(\Theta)$ from 0° to 720° . Similarly, repeat this with radian measure for the domain 0 to 4π .

Unit 2: Sinusoidal Functions—Suggestions for Teaching and Learning

- Discuss with students a comparison between a variety of graphs where some graphs are periodic but not sinusoidal and others are periodic and sinusoidal. Comparing the graphs students should be able to conclude that all sinusoidal functions are periodic but not all periodic functions are sinusoidal.
- Discuss with students how to use graphing technology to examine the effects of manipulating the value of the variable a by comparing the sinusoidal functions $y=\sin(x)$ and $y=a\sin(x)$ where $a>0$. The graph can be compared by comparing the amplitude if $a<0$, examining how the shape is affected by the parameter a , and considering the value of a , affects the cosine graph in the same way that it affects the sine graph.
- Ensure that students understand how to match a sinusoidal graph with its corresponding equation.
- Ensure that students understand how to find a sinusoidal regression. Regression analysis can be found using a variety of techniques such as graphing calculators.

Unit 2: Sinusoidal Functions —Suggestions for Assessment

- Instructors can use the BLM's on the CD-ROM to further reinforce the unit concepts.
- The BLM's on the CD-ROM can be useful for developing unit tests and the final exam.
- Instructors have discretion to combine the last unit test with the final exam if beneficial to the student.
- Students must pass the final exam with a minimum grade of 50% to receive credit for this course.
- Instructors should encourage students to reflect on the math concepts in this unit to relate to everyday life.
- Instructors should engage students in discussions to verbalize student thinking on the math concepts.
- Instructors should require students to always show complete calculations with correct units when relevant.
- Instructors can use their own professional judgment to design assessment tools (additional exercises, word problems, assignments, reflections, math journals, etc.) to meet individual student needs.

Unit 3: Financial Mathematics: Borrowing Money—Suggestions for Teaching and Learning

- Discuss with students why financial literacy is a crucial life skill. Students should realize why developing the set of knowledge that allows them to make appropriate decisions when managing personal finances is important; e.g., students should be knowledgeable about compound interest and the costs and benefits of renting, leasing & buying.
- Discuss with students that while compound interest is beneficial to the person receiving the interest, it costs the person paying the interest on an loan, mortgage, or credit card a significant amount of money.
- Discuss examples when one might have to pay simple interest and examples for compound interest.
- Discuss with students that many products offered by financial institutions typically change compound interest.
- Ensure that students understand the formulas used to find simple interest and compound interest.
- Ensure that students understand that simple interest increases linearly, whereas compound interest increases exponentially. Students should also understand that 3% interest “compounded monthly” does not mean 3% per month; it means .25% per month.
- Discuss with students bi-weekly payments, accelerated bi-weekly payments and semi-monthly payments.

Unit 3: Financial Mathematics: Borrowing Money—Suggestions for Teaching and Learning

- Ensure that students understand how to calculate the total interest paid for different compounding periods, given the same initial principal, interest rate and term.
- Discuss with students that when making financial decisions, it is important for them to understand the rate of interest charged, as well as the compounding, as these can create large differences over long periods of time.
- Discuss with students that interest accumulates faster when there is an increase in the frequency of compounding.
- Ensure that students understand the effects that changing a variable or multiple variables has on the total cost of the loan.
- Discuss with the students that decreasing the amortization paid on a mortgage results in increasing the monthly payments but decreases their total interest paid.
- Ensure that students understand the variables in the compound interest question: present value, regular payment amount, payment frequency, number of payments, annual interest rate, compounding frequency, total interest and future value.
- Discuss with students the advantages and disadvantages of using line of credit, in-store finance options and credit cards for purchase.
- Discuss with student decisions about whether to buy, rent or lease in a variety of contexts.
- Discuss with students situations where assets can appreciate or depreciate and the factors which contribute to both.
- Discuss with students why they should factor in depreciation when deciding to buy a new vehicle.
- Discuss with students why completing a cost-and-benefit analysis is essential when deciding to rent, lease or buy.

Unit 3: Financial Mathematics: Borrowing Money —Suggestions for Assessment

- Instructors can use the BLM's on the CD-ROM to further reinforce the unit concepts.
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